

The isospin independent spin-orbit force in the extended Skyrme model

Abdellatif Abada *

Laboratoire de Physique Nucléaire, CNRS/IN2P3 Université de Nantes,
2, Rue de la Houssinière, 44072 Nantes Cedex 03, France

Abstract

By using the product ansatz as an approximation for the two-baryon system we investigate the isoscalar nucleon-nucleon spin-orbit potential in an extended Skyrme model including both fourth- and sixth-order terms. As it is the case for the Skyrme model, we still obtain the wrong sign for this interaction. Nevertheless, concerning the order of magnitude, the extended Skyrme model provides a better agreement with phenomenological potentials, as compared to the standard one.

PACS numbers : 11.10.Lm, 11.10Ef, 14.20Gk

LPN 94-01

February 1994

Submitted to Phys. Lett. B

*email : "ABADA@nanvs2.in2p3.fr"

The Skyrme model [1], recognized as the simplest chiral realization of QCD at low energy and large N_c [2], has been extensively used to describe static and dynamical properties of baryons. However, it has been shown recently [3, 4], through the study of some properties of the nucleon, that one should not restrict oneself to the standard Skyrme model for the description of low-energy hadron physics, but consider extensions of this model including higher order terms in powers of the derivatives of the pion field. In this letter we wish to study the effects of such additional terms on the dynamical part of the nucleon-nucleon interaction. We examine the influence of the sixth-order term generated by ω -meson exchange [5] on the nucleon-nucleon spin-orbit force, where we focus on the isospin independent part of the interaction. Several attempts to extract the isoscalar N - N spin-orbit interaction from the standard Skyrme model have given the wrong sign [6, 7]. Riska and Schwesinger [8] claimed that the inclusion of the sixth-order term leads to the right sign. However, these authors used a value for the corresponding parameter which is in conflict with the experimental $\omega \rightarrow \pi\gamma$ width.

1. The effective Lagrangian density corresponding to the extended Skyrme model we use can be expressed in terms of an $SU(2)$ matrix field U which characterizes the pion field. It reads

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32e^2} \text{Tr} \{ [(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2 \} - \frac{1}{2} \frac{\beta_\omega}{m_\omega^2} B_\mu B^\mu + \frac{f_\pi^2}{4} m_\pi^2 \text{Tr} (U + U^\dagger - 2) \quad (1)$$

where B_μ is the baryon current [1]

$$B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr} \{ (\partial_\nu U)U^\dagger (\partial_\alpha U)U^\dagger (\partial_\beta U)U^\dagger \}. \quad (2)$$

The first term in Eq. (1) corresponds to the nonlinear σ -model, f_π being the pion decay constant. The second term, which is of fourth-order in powers of derivatives of the pion field and parametrized by the dimensionless factor e , was introduced by Skyrme in order to stabilize the soliton. The third term is of order six in the derivatives and can be derived from a local approximation of an effective model with ω mesons [5]. The constant m_ω is the ω -meson mass (782 MeV) while β_ω is a dimensionless parameter related to the $\omega \rightarrow \pi\gamma$ width. The last term in Eq. (1) which is proportional to the square of the pion mass m_π (139 MeV) implements a small explicit breaking of chiral symmetry.

For the system of two interacting solitons we use the product ansatz as suggested by Skyrme [9]. We also introduce rotational dynamics to obtain the appropriate spin and isospin structure [10]. Thus,

the field configuration of the two-nucleon system separated by a vector \mathbf{r} reads:

$$U_{B=2} \equiv U_2(\mathbf{x}, \mathbf{r}, A, B) = AU_H(\mathbf{x} - \mathbf{r}/2)CU_H(\mathbf{x} + \mathbf{r}/2)B^+ , \quad (3)$$

where $C = A^+B$ and A and B are $SU(2)$ matrices. To carry out a simultaneous quantization of the relative motion of the two nucleons and the rotational motion we need to treat \mathbf{r} , A and B as collective coordinates. Hence we make all these parameters (\mathbf{r}, A, B) time dependent. In Eq. (3), U_H is the commonly used $SU(2)$ matrix for a single soliton with the hedgehog ansatz:

$$U_H(\mathbf{x}) = \exp[i\vec{\tau} \cdot \hat{\mathbf{x}} F(|\mathbf{x}|)] , \quad (4)$$

where $F(|\mathbf{x}|)$ obeys the usual boundary conditions for winding number one, and the τ_a 's are the Pauli matrices. The notation $\hat{\mathbf{x}}$ means $\mathbf{x}/|\mathbf{x}|$. We have to note that the product configuration (3) is an approximation which has inconveniences and advantages [11]. We choose it for its relative simplicity as compared to other two-baryon field configurations which can be found in the literature. The region of validity of the product ansatz corresponds to a separation r much larger than 1 fm.

2. The spin-orbit potential will emerge due to a coupling between the relative motion and the spins of the two nucleons. So we have to calculate the kinetic energy

$$K = \int d^3x \mathcal{K}(U_2) , \quad (5)$$

where $\mathcal{K}(U_2)$ is that part of the Lagrangian (1) involving only quadratic time derivatives, in which U is given by Eq. (3). The full expression of Eq. (5) with respect to (\mathbf{r}, A, B) and their derivatives is very complicated and contains several terms which do not need to be written in this letter. They will be reported on a forthcoming publication [12]. The isospin independent part K_0 of the kinetic energy (5) reads

$$K_0 = \frac{1}{2} \dot{\mathbf{Q}} \mathbf{M}_0 \dot{\mathbf{Q}} , \quad (6)$$

where

$$\dot{\mathbf{Q}} = (\dot{\mathbf{r}}, \mathbf{w}_+, \mathbf{w}_-) . \quad (7)$$

In Eq. (7), \mathbf{w}_\pm are the sum and the difference of the rotational velocities respectively:

$$\mathbf{w}_\pm = -\frac{i}{2} \left(\text{Tr}(\vec{\tau} A^+ \dot{A}) \pm \text{Tr}(\vec{\tau} B^+ \dot{B}) \right) ,$$

and \mathbf{M}_0 is the 9×9 mass matrix

$$\mathbf{M}_0 = \begin{pmatrix} \alpha(r) I & -\beta(r) \not{e} & 0 \\ \beta(r) \not{e} & \gamma(r) I & 0 \\ 0 & 0 & \gamma(r) I \end{pmatrix} \quad (8)$$

where I is the 3×3 identity matrix and $\not{e}_{ij} \equiv \epsilon_{ijk} \hat{\mathbf{r}}_k$. The functions α, β and γ depend only on the relative distance $r = |\mathbf{r}|$ and are determined from the chiral function F (see Eq. (4)). We have to mention that the mass matrix \mathbf{M}_0 contains other terms, such as a coupling between rotational velocities or a radial motion term, which contribute in principle to K_0 . However, we find these terms very small and thus neglect them [7, 12]. In this sense, the matrix \mathbf{M}_0 given by Eq. (8) should be considered as the dominant contribution to the isospin independent kinetic energy (6).

Before identifying and extracting the spin-orbit potential from Eq. (6) one has to treat carefully the conversion from velocities to canonical momenta. Indeed, one has to invert properly the mass matrix \mathbf{M}_0 in order to move from a Lagrangian formalism to a Hamiltonian one. In terms of the canonical momenta

$$\mathbf{P} = \mathbf{M}_0 \dot{\mathbf{Q}} = (\mathbf{p}_r, \mathbf{s}, \mathbf{s}_-) ,$$

where \mathbf{p}_r is the conjugate momentum to \mathbf{r} , and $\mathbf{s} = \frac{\vec{\sigma}_1}{2} + \frac{\vec{\sigma}_2}{2}$, $\mathbf{s}_- = \frac{\vec{\sigma}_1}{2} - \frac{\vec{\sigma}_2}{2}$ the conjugate momenta to \mathbf{w}_+ , \mathbf{w}_- respectively [10, 13] ($\vec{\sigma}_i/2$ being the spin of the nucleon i , $i = 1, 2$), the kinetic energy (6) becomes:

$$K_0 = \frac{1}{2} \mathbf{P} \mathbf{M}_0^{-1} \mathbf{P} . \quad (9)$$

It is straightforward to invert the mass matrix (8). Its expression reads:

$$\mathbf{M}_0^{-1} = \frac{1}{\alpha\gamma - \beta^2} \left[\begin{pmatrix} \gamma I & \beta \not{e} & 0 \\ -\beta \not{e} & \alpha I & 0 \\ 0 & 0 & (\alpha\gamma - \beta^2)/\gamma I \end{pmatrix} - \frac{\beta^2}{\alpha\gamma} \begin{pmatrix} \gamma \not{\mathbf{r}} & 0 & 0 \\ 0 & \alpha \not{\mathbf{r}} & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \quad (10)$$

where \not{e} has been defined above and $\not{\mathbf{r}}_{ij} \equiv \hat{\mathbf{r}}_i \hat{\mathbf{r}}_j$. By developing Eq. (9) we obtain

$$2K_0 = \frac{1}{\alpha\gamma - \beta^2} \left(\gamma \mathbf{p}_r^2 + \alpha \mathbf{s}^2 + \frac{2\beta}{r} \mathbf{l} \cdot \mathbf{s} - \frac{\beta^2}{\alpha} (\mathbf{p}_r \cdot \hat{\mathbf{r}})^2 - \frac{\beta^2}{\gamma} (\mathbf{s} \cdot \hat{\mathbf{r}})^2 \right) + \frac{1}{\gamma} \mathbf{s}_- \cdot \mathbf{s}_- \quad (11)$$

where $\mathbf{l} = \mathbf{r} \times \mathbf{p}_r$ is the angular momentum. Now we can easily extract from Eq. (11) the isoscalar spin-orbit force with the result:

$$V_{ls}(r) = + \frac{\beta}{r (\alpha\gamma - \beta^2)} . \quad (12)$$

3. In order to compute the spin-orbit potential (12) we have to take a set of parameters of the model, namely, f_π, e and β_ω . The physically more sensible manner to choose the parameters is to remember that the Skyrme model involves only meson fields, and where baryons emerge as topological solitons. Therefore, since it is a meson theory, the parameters of the model have to be fixed by fitting to the low-energy *meson* observables and not to baryon data as it has been done in Ref. [7]. The experiment yields $f_\pi = 93$ MeV for the pion decay constant. The dimensionless parameter e can be determined from chiral perturbation theory [14]. The best set of chiral low-energy constants has been determined by Riggenbach *et al* [15], in analyzing the K_{l4} decays. As a consequence, one finds $e = 7.1 \pm 1.2$ [3]. Finally, the last parameter β_ω is obtained by fitting to the $\omega \rightarrow \pi\gamma$ width, yielding $\beta_\omega = 9.3$ [16]. We have to mention that this set of parameters yields values of the nucleon mass (including the Casimir effect), the Δ - N mass splitting, the axial-vector coupling constant, the Roper resonance, etc., close to the experimental situation [3, 4].

Our first result is that the isoscalar spin-orbit force obtained with these parameters is *repulsive* and thus has the wrong sign as compared to the experimental situation. This disappointing prediction of Skyrme models is still not explained until now. In order to compare our results to the data we consider therefore $-V_{ls}$ instead of V_{ls} (c.f., Eq. (12)). Of course one can force the interaction to be attractive by increasing the value of the parameter β_ω and thus the sixth-order term dominates the second- and fourth-order terms. This was done by the authors of Ref. [8] who used a value of $\beta_\omega = 22.4$. However, this way to proceed is not acceptable since this value of β_ω disagrees stongly with the experimental $\omega \rightarrow \pi\gamma$ width.

We plot in Fig. 1 the behaviour of $-V_{ls}$ (12) with respect to the nucleon-nucleon separation r for $[f_\pi = 93 \text{ MeV}, e = 7.1, \beta_\omega = 9.3]$ and compare it to the standard Skyrme model prediction ($\beta_\omega = 0$). We also plot in the same figure the corresponding part of the phenomenological Bonn potential [17]. From Fig. 1 we obviously see that the inclusion of the sixth-order term in the Lagrangian (1) improves considerably the prediction of the isoscalar spin-orbit force in the region of validity of the model, i.e., $r \geq 1$ fm. It is expected that the agreement can not be perfect due to several reasons. One of them is the use of the approximation of the product ansatz (3) to describe two-interacting nucleons. A second reason is that the model used here (1) should be regarded as a minimal extension of the Skyrme model. Concerning the region of smaller r (not shown on the figure) the prediction of the model differs completely from the phenomenological potential because this region corresponds to processes

of large momentum transfers where one should not use effective Lagrangians but perturbative QCD.

4. In this letter we have derived the isospin independent spin-orbit interaction from the extended Skyrme model including, in addition to the nonlinear σ model, fourth- and sixth-order terms in powers of the derivatives of the pion field. Concerning the order of magnitude, we find a considerable improvement as compared to the Skyrme model prediction and thus, confirm the conclusions of Refs. [3, 4]. Indeed, the standard Skyrme model is insufficient for an accurate description of baryon phenomenology and one should generalize this model by including higher order terms in the chiral Lagrangian. In spite of that, our major result is that, even with the sixth-order term present, we still get the *wrong* sign for this force. One should try to understand why the standard Skyrme model as well as the extended one (1) give the wrong sign and not mask the problem by considering unrealistic values of the parameters [8]. This difficulty with the isoscalar spin-orbit force seems to be similar to that of missing central attraction. The problem concerning the lack of attraction in the central potential has been solved by considering effective Lagrangians which incorporate low mass mesons with finite mass (the scalar meson having an important role) [18]. Maybe one has to investigate in this way in order to correct the anomaly of that sign [19].

Acknowledgments

We are very grateful to D. Kalafatis, B. Loiseau, B. Moussallam, D. O. Riska, R. Vinh Mau and N. R. Walet for helpful discussions and criticisms and express our appreciation to D. Kalafatis and N. R. Walet for a critical reading of the manuscript.

References

- [1] T. H. R. Skyrme, Proc. Roy. Soc. **A260** (1961) 127.
- [2] E. Witten, Nucl. Phys. **B223** (1983) 422 and 433.
- [3] B. Moussallam, Ann. Phys. (N.Y.) **225** (1993) 264.
- [4] A. Abada and H. Merabet, Phys. Rev. D **48** (1993) 2337.
- [5] A. Jackson, A. D. Jackson, A. S. Goldhaber, G. E. Brown and L. C. Castillejo, Phys. Lett. **154B** (1985) 101.

- [6] E. M. Nyman and D. O. Riska, Phys. Lett. B **175** (1986) 392 ; **183** (1987) 7 ; D. O. Riska and K. Dannbom, Phys. Scr. **37** (1988) 7 ; T. Otofujii *et al*, Phys. Lett. B **205** (1988) 145.
- [7] R. D. Amado, B. Shao and N. R. Walet, Phys. Lett. B **314** (1993) 159 and an erratum to be published in Phys. Lett. B ; Phys. Rev. C **48** (1993) 2498 and an erratum to be published in Phys. Rev. C ; N. R. Walet, private communication.
- [8] D. O. Riska and B. Schwesinger, Phys. Lett. B **229** (1989) 339.
- [9] T. H. R. Skyrme, Nucl. Phys. **31** (1962) 556.
- [10] G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. **B228** (1983) 552.
- [11] A. Jackson, A. D. Jackson and V. Pasquier, Nucl. Phys. A **432** (1985) 567.
- [12] A. Abada, in preparation.
- [13] E. M. Nyman, Phys. Lett. B **162** (1985) 244.
- [14] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y) **158** (1984) 142.
- [15] C. Riggenbach, J. Gasser, J. F. Donoghue and B. R. Holstein, Phys. Rev. D **43** (1991) 127.
- [16] M. Lacombe, B. Loiseau, R. Vinh Mau and W. N. Cottingham, Phys. Rev. D **38** (1988) 1491.
- [17] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. **149** (1987) 1.
- [18] D. Kalafatis and R. Vinh Mau, Phys. Rev. D **46** (1992) 3906 and references therein.
- [19] R. Vinh Mau, private communication.

Figure captions

FIG. 1. The isoscalar spin-orbit potential (12) with a minus sign in the case where the parameters fit to low energy *meson* data. The dotted line corresponds to the Skyrme model (i.e., $\beta_\omega = 0$), while the full one corresponds to the extended model with $\beta_\omega = 9.3$. The dashed line is the corresponding term in the Bonn potential [17].

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9401341v2>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9401341v2>